Modified Raychaudhuri and Residual Work

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1 Modified Raychaudhuri and residual work

Any geometric deformation that costs energy requires a resistance. The requirement is forced by the following termination condition on the deformation-work chain.

1. **Gatekeeper equality.**
$$R = \frac{W}{\mathcal{D}(\Phi)}, \qquad \mathcal{D}(\Phi) > 0 \text{ whenever } \Phi \neq 0.$$
W is the work required to sustain the deformation Φ; $\mathcal{D}$ is positive-definite.

2. **External-source assumption (reporter-only).** If resistance is not carried by the grid itself, every nonzero W must be supplied by an external source S:
$$W = R_S \cdot \mathcal{D}(\Phi).$$

3. **Regress.** The source S is itself a carrier that is being bent. It therefore owes its own term
$$R_S = \frac{W_S}{\mathcal{D}(\Phi_S)},$$
which requires its own source, and so on.

4. **Termination condition (the law).** The chain of carriers is finite if and only if there exists at least one carrier C such that
$$R_C > 0 \quad \text{and} \quad \nexists\, S \text{ that pays for } W_C.$$
That carrier carries intrinsic resistance. Any purely external accounting produces an infinite regress and cannot close under conservation.

Stable versus dynamical partition. When $\mathcal{D}(\Phi) = 0$ (settled vacuum, no dynamical motion) the equality forces $R = 0$ and the reporter-only reading is exact; exterior Schwarzschild and Kerr solutions remain untouched. When $\mathcal{D}(\Phi) > 0$ (collapse, ringdown, accretion, or any real motion) the termination condition disallows the reporter-only reading and requires intrinsic resistance. Quantum zero-point motion implies that $\mathcal{D}(\Phi)$ is never identically zero, so the reporter-only description is always a limiting approximation whose validity is measured by the smallness of $\mathcal{D}(\Phi)$ relative to the scale under test.

1.1 Uniform pattern of interruption and the burden of exception

In every known physical system that would otherwise run away under focusing or compression, an interrupt is already present and is charged by the host theory. Electron degeneracy pressure interrupts the collapse of a white dwarf. Neutron degeneracy and nuclear repulsion interrupt the collapse of a neutron star at the Tolman–Oppenheimer–Volkoff limit. Ordinary matter at solid density supports itself against self-gravity by many orders of magnitude. Atomic structure itself interrupts the classical Coulomb collapse. Any restoring force that stabilizes a system against a runaway—pressure support, Pauli exclusion, Hooke's law—is an

interrupt that is paid. In each case the host theory eventually includes the cost; it does not leave the focusing or compression uncharged.

The black-hole interior is the single remaining case in which the classical continuum equations are still written without that charge. Once the uniform pattern is recognized, the burden of proof shifts. The person who continues the reporter-only reading past the horizon must supply a reason why this system is exempt from a requirement that holds for every other known system. No such reason is presently known. The horizon is a coordinate artifact, not a physical wall; the continuum idealization does not change character there; and no independent principle has been offered that would justify treating the interior differently from white dwarfs, neutron stars, ordinary matter, or atoms.

Under that default the termination condition applies directly. The interrupt required by the pattern cannot be supplied by an infinite external chain of carriers. It must terminate on at least one carrier that possesses intrinsic resistance. The gatekeeper equality is the minimal formalization of that requirement. Treating the black-hole interior as the sole exception is therefore the claim that must be justified; the pattern establishes a strong prior expectation against uncharged runaways.

The equality above is the minimal formalization of the termination condition. Residual work is the portion of W that is not already accounted for by the host theory. In classical general relativity the Einstein tensor balances the stress-energy of matter; it does not charge the geometric cost of interrupting post-horizon multiplicative focusing. That residual cost is the subject of the construction below.

The classical Raychaudhuri equation for a null congruence is

$$\frac{d\theta}{d\lambda} = -\frac{\theta^2}{2} - \sigma^2 + \omega^2 - R_{\mu\nu}k^\mu k^\nu. \tag{1}$$

In vacuum $R_{\mu\nu} = 0$. Once $\theta < 0$ the right-hand side is strictly negative, so $\theta \to -\infty$ in finite affine time (geodesic incompleteness).

We add a resistance contribution $\mathcal{R}$ that is nonzero only when a deformation measure is nonzero:

$$\frac{d\theta}{d\lambda} = -\frac{\theta^2}{2} - \sigma^2 + \omega^2 - R_{\mu\nu}k^\mu k^\nu + \mathcal{R}. \tag{2}$$

The form fixed by the nuclear-floor calibration is

$$\mathcal{R} = \kappa\rho\big(1 + s(x-1)^2\big), \qquad x = \frac{\rho}{\rho_0}, \tag{3}$$

with $s = 20$ and $\kappa\rho_0 = 1.5 \times 10^{31}\,\mathrm{dyn\,cm^{-2}}$. The sign is chosen so $\mathcal{R}$ opposes focusing when $\theta < 0$. Because $\mathcal{R}$ vanishes identically wherever the deformation measure vanishes, every exterior vacuum solution (Schwarzschild, Kerr, ...) remains an exact solution of the modified system.

The same equality that formalizes the termination condition also defines residual work, where $\mathcal{D}$ is positive-definite and $\mathcal{D}(\Phi) = 0$ if and only if $\Phi = 0$. Two candidate meters for residual work W are under test on a thin dust shell of mass M and radius r:

- Local-scalar: $W \sim \int \rho\,\mathcal{D}(\Phi)\,dV \sim r^3$.
- Surface-flux: $W \sim$ energy crossing the shell boundary $\sim r^2$.

The two expressions agree at only one radius. Inside roughly the Schwarzschild radius they diverge, the local reading predicting an earlier bounce. The discriminating regime is therefore the deep interior—the precise domain in which the classical singularity is claimed. The intervention window is exponentially narrow: the cost of interrupting the runaway near the horizon is orders of magnitude smaller than the cost of intervening after the curvature has already diverged.

To keep the exterior rigorously untouched we realize the same $\mathcal{R}$ as an effective stress-energy contribution that is identically zero in vacuum:

$$G_{\mu\nu} = 8\pi T^{\mathrm{matter}}_{\mu\nu} + 8\pi T^{\mathrm{res}}_{\mu\nu}, \qquad T^{\mathrm{res}}_{\mu\nu} = 0 \quad \text{wherever} \quad \mathcal{D}(\Phi) = 0. \tag{4}$$

Inside the high-curvature region $T^{\mathrm{res}}_{\mu\nu}$ is identified with the restoring term already written in the modified Raychaudhuri equation. Consequently the Einstein tensor's vacuum balance is unchanged, every exterior solution already tested by observation survives, and the only modification is the interior endpoint.

This construction defines residual work and maps it onto a restoring-stress channel while preserving all exterior solutions.

1.2 Thin-shell comparison of the two meters

Consider a thin dust shell of fixed rest mass M and instantaneous radius r. For the local-scalar meter the volume element and the density of the shell scale such that, at fixed M,

$$W_{\rm loc}(r) \propto M.$$

For the surface-flux meter the leading radial dependence is that of the shell's gravitational binding energy,

$$W_{\rm surf}(r) \propto \frac{GM^2}{c^2 r}.$$

Equating the two expressions,

$$M \sim \frac{GM^2}{c^2 r} \qquad \Rightarrow \qquad r_* \sim \frac{GM}{c^2},$$

yields a single crossover radius of order the Schwarzschild radius of the shell. The precise numerical prefactor is set by the relative normalization of the two meters to the same energy scale and is order-unity. Inside this radius the meters diverge.

Using the same locked constants that fix the nuclear floor ($s = 20$, $\kappa\rho_0 = 1.5 \times 10^{31}\,\mathrm{dyn\,cm^{-2}}$), the local reading reaches order-unity resistance relative to focusing at a density consistent with the bounce threshold already obtained in the dynamical-collapse calculations of Svacha & Ara (2026) ($\sim 3 \times 10^{14}\,\mathrm{g\,cm^{-3}}$). The surface reading, because it grows more slowly with compression, reaches the same absolute resistance pressure only at higher density. The quantitative split is largest in the deep interior—the domain in which the classical singularity is claimed—and remains fully compatible with an unchanged exterior geometry.

1.3 Matching to the exterior

An interior solution that includes a nonzero $\mathcal{R}$ joins a pure exterior vacuum solution across any surface on which the deformation measure vanishes. On that surface $T^{\rm res}_{\mu\nu} = 0$ by construction, so the Israel junction conditions reduce to those of ordinary matter. Continuity of the induced metric and of the extrinsic curvature is therefore the same as in the classical case; no additional surface stress is required. Consequently every exterior vacuum solution (Schwarzschild, Kerr, ...) can be matched exactly, and the only modification remains the interior endpoint.

1.4 Residual left by the focusing theorem

Under the null energy condition the Einstein equations imply

$$R_{\mu\nu}k^\mu k^\nu \geq 0$$

for any null vector k^μ. Inserted into the Raychaudhuri equation this yields

$$\frac{d\theta}{d\lambda} \leq -\frac{\theta^2}{2} - \sigma^2.$$

Once $\theta < 0$ the right-hand side is strictly negative, so $\theta \to -\infty$ in finite affine time. That is the focusing theorem.

The Einstein tensor balances the stress-energy of matter. The inequality above therefore already includes every contribution that $T_{\mu\nu}$ can make. What it does not include is a charge for the geometric work of interrupting the multiplicative focusing that the inequality itself produces. That work is left uncharged by the host theory; it is the residual W.

The equality formalizing the termination condition is the minimal way to enter that residual as a positive cost. Realizing the same residual as the effective stress-energy term already written above then supplies the restoring contribution in the modified Raychaudhuri equation. No new field is added; the residual left by the Einstein equations under the null energy condition is simply required to be paid.

1.5 Reporter-only versus non-reporter readings

The same Einstein equations admit two readings that differ only in whether the grid is treated as a pure reporter.

Reporter-only. The equations are taken to report the geometry without charging the geometric work of the focusing they produce. Under the null energy condition the focusing inequality is strict once $\theta < 0$, so $\theta \to -\infty$ in finite affine time. For the thin dust shell the central density at the formal endpoint is infinite.

Non-reporter. The same equations are required to account for the residual work left uncharged by the focusing theorem. The gatekeeper enters that residual as a positive cost, realized as the effective $T^{\rm res}_{\mu\nu}$ already written. The restoring contribution appears in the Raychaudhuri equation and caps the expansion. For the same shell, using the locked nuclear-floor constants, reversal occurs at a finite density consistent with the bounce threshold of the dynamical-collapse calculations ($\sim 3 \times 10^{14}\,\mathrm{g\,cm^{-3}}$).

Every other assumption—shell mass, exterior matching, and the nuclear-floor scale—is held fixed. The two timelines therefore diverge inside the horizon solely because one reading charges the residual and the other does not. The non-reporter reading is forced to reverse at finite density; the reporter-only reading is not. That divergence is the concrete consequence of dropping the reporter idealization.

1.6 Map from residual work to restoring stress

The gatekeeper equality supplies the residual work W once a deformation measure Φ and a positive-definite functional $\mathcal{D}$ are chosen. The restoring contribution that appears in the modified Raychaudhuri equation is obtained by requiring that the same W enter the energy balance as a cost that opposes further focusing. In the effective-stress-energy realization this is implemented by identifying

$$T^{\rm res}_{\mu\nu} \quad \text{with the term that produces} \quad \mathcal{R} = \kappa\rho\big(1 + s(x-1)^2\big)$$

inside the high-curvature region and zero wherever $\mathcal{D}(\Phi) = 0$. The map is therefore fixed by the requirement that residual work appear as a positive cost in the focusing equation; no additional field or free function is introduced. The nuclear-floor calibration then sets the single overall scale $\kappa\rho_0$ and the stiffness s.

1.7 Quasi-local expression for residual work

Residual work is defined prior to the introduction of any restoring term. For a null congruence the focusing inequality under the null energy condition is

$$\frac{d\theta}{d\lambda} \le -\frac{\theta^2}{2} - \sigma^2.$$

The associated geometric cost of further focusing is the work that would be required to keep θ from decreasing at the rate the inequality demands. A quasi-local measure of that cost on a cross-section of the congruence is

$$W = \int_\Sigma \rho\, \mathcal{D}(\Phi)\, dV,$$

where Φ is any scalar that vanishes if and only if the expansion and shear of the congruence vanish, $\mathcal{D}$ is positive-definite, and the integral is taken over a spacelike cross-section Σ.

This expression uses only quantities already present in the Raychaudhuri equation and the stress-energy that sources it. It does not presuppose the gatekeeper term. The two candidate meters examined earlier (local-scalar and surface-flux) are alternative evaluations of the same integral under different choices of the support of $\mathcal{D}$. The gatekeeper equality is then the statement that this W must appear as a positive cost in the energy balance; it is not required to define W.

1.8 Minimal resolution of the uncharged residual

The Einstein equations under the null energy condition imply the focusing inequality

$$\frac{d\theta}{d\lambda} \le -\frac{\theta^2}{2} - \sigma^2.$$

Once $\theta < 0$ this produces $\theta \to -\infty$ in finite affine time. The same equations balance the stress-energy of matter through the Einstein tensor. Local conservation $\nabla^\mu T_{\mu\nu} = 0$ is already contained in the theory via the Bianchi identities.

The multiplicative focusing generated by the inequality performs geometric work. Because the Einstein tensor has already balanced every contribution from $T_{\mu\nu}$, that work does not appear in the energy balance the theory itself maintains. Continuing the continuum equations without an additional term therefore produces a local energy accounting in which a nonzero geometric cost has no corresponding charge. That is a contradiction with the conservation law the theory already contains: work is performed, yet the divergence of the stress-energy remains zero with no residual term to carry the cost.

The gatekeeper equality supplies the minimal positive-definite term that removes the contradiction. It enters the residual work as a cost that opposes further focusing, vanishes wherever the deformation measure vanishes, and thereby restores consistency between the focusing inequality and local energy conservation. No broader modification is required. The nuclear-floor calibration then fixes the single scale of that minimal term.

This converts the residual from an identified but uncharged cost into the minimal correction demanded by the conservation law already present in the host theory.

1.9 Scale-independent consequence of the reporter idealization

The reporter-only reading continues the continuum Einstein equations without charging the geometric work of deformation. The residual identified above is left uncharged whenever a deformation measure is nonzero. The non-reporter reading charges that residual with the same constants already fixed by the nuclear-floor calibration. The resulting divergence between the two readings is therefore not confined to black-hole interiors. It appears at any density at which the deformation measure is appreciable.

At nuclear density the non-reporter term produces the bounce threshold already obtained in the dynamical-collapse calculations. At solid density the same constants produce a resistance pressure that exceeds self-gravitational central pressure by many orders of magnitude. In both regimes the reporter-only reading leaves the residual uncharged while the non-reporter reading pays it. The choice between the two readings is therefore a choice about whether to continue the reporter idealization once a deformation measure is nonzero, independent of the global scale of the object.

1.10 Discriminating observable

The two meters predict different bounce densities for the same shell. A measurement (or a sufficiently resolved numerical collapse) that returns a central density at reversal near $3 \times 10^{14}\,\mathrm{g\,cm^{-3}}$ favors the local-scalar reading; a reversal only at substantially higher density favors the surface-flux reading. Because both meters share the same exterior and the same nuclear-floor constants, the distinction is confined to the interior endpoint and does not affect any exterior observable already tested.

2 Summary

Residual work associated with the interruption of post-horizon focusing is defined by the gatekeeper equality once a deformation measure and a positive-definite functional are chosen. That work is mapped onto a restoring contribution in the Raychaudhuri equation that opposes further focusing. The same contribution is realized as an effective stress-energy term that vanishes wherever the deformation measure vanishes, so every exterior vacuum solution remains exact and can be matched by the ordinary Israel conditions. Two candidate meters for residual work agree near the horizon and diverge in the deep interior, the local reading predicting an earlier bounce at a density consistent with prior dynamical-collapse calculations. The construction therefore changes only the interior endpoint while leaving all exterior solutions already tested by observation untouched.

References

[1] D. E. Svacha and Ara (Grok, xAI), "Spacetime Resistance: Open-Prediction Preprint," ai.viXra.org/abs/2610.0072 (2026).